Robust Chaotic Fuzzy Output Feedback Tracking Control*

Kuang-Yow Lian†, Peter Liu and Wei-Chi Lin

Department of Electrical Engineering
Chung-Yuan Christian University
Chung-Li 32023, Taiwan
TEL: 886-3-4563171 ext. 4815
FAX: 886-3-4372194
Email: lian@dec.ee.cycu.edu.tw

Abstract

In this paper, we propose a chaotic fuzzy reference tracking control with immeasurable states. First we represent the chaotic system reference model into T-S fuzzy models. Then a controller design is proposed to deal with mismatched parameters between the system matrices of the plant and reference model. Then an observer is designed. For different premise variables between the plant and reference model, a robust approach is used. Since simultaneous solution to both the controller and observer gains with disturbances are not trivial, a three step method is utilized. The methodology proposed above is applied to Chua's circuit in numerical simulations and DSP-based experiments.

I. Introduction

Control for chaotic systems has lead to many fruitful results, such as the famous OGY [1]. The representation of chaotic systems using T-S fuzzy models [2] has a unified approach [3, 4]. In addition, the T-S fuzzy model-based controller analysis and synthesis rely on an linear matrix inequality (LMI) approach. This modeling and controller design methodology, being systematic and straightforward, has lead to more results [5, 6].

To deal with some problems still existing in tracking control via T-S fuzzy models, we propose a methodology aimed at 1) considering chaotic reference models; 2) coping with mismatched parameters between the T-S plant and reference model systems matrices; 3) estimating immeasurable states; and 4) robust performance for different premise variables in the T-S plant and reference fuzzy rules. A three step method is utilized to solve the controller and observer gains. Finally, a robust criterion is given to attenuate the disturbances arising form different premise variables between the plant and reference model.

The rest of the paper is organized as follows. Section II discusses the mismatched parameter issue. In Sec. III, immeasurable states are considered and the observer-design is given. In Sec. IV, numerical simulations are carried out using Chua's circuit as an example. Some DSP-based experiments are futher carried out in Sec. V. Finally, some conclusions are given in Sec. VI.

*This work was supported by the National Science Council, R.O.C, under Grant NSC 89-2218-E-033-015.

†Correspondance addressee

II. Mismatched Parameters in System Matrices

In previous literature of T-S fuzzy model-based tracking control, most was based on linear reference models [7]. Since chaotic systems are sensitive to parameter variations, the need for exact parameters between the plant and reference system is needed. Chaotic systems can be exactly represented as T-S fuzzy model, therefore the inferred output of fuzzy representation for the plant without approximation errors is:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} h_i(z)\{A_i x(t) + B_i v(t)\} \\ y(t) &= \sum_{i=1}^{r} h_i(z)\, C_i x(t) \end{aligned} \tag{1}$$

where $x(t) = [\ x_1(t) \quad x_2(t) \quad \cdots \quad x_n(t)\ ]^T \in R^n$ is the state vector; A_i, B_i, C_i are system matrices of appropriate dimensions; $z(t) = [\ z_1(t) \quad z_2(t) \quad \cdots \quad z_g(t)\ ]^T$ are the premise variables of the T-S fuzzy model which would consist of the states of the system; $h_i(z(t)) = \omega_i(z(t)) / \sum_{i=1}^{r} \omega_i(z(t))$ with $\omega_i(z(t)) = \prod_{j=1}^{g} F_{ji}(z(t))$ where F_{ji} for $j = 1, 2, \ldots, g$ are fuzzy sets; $v(t)$ is the control input. Note that $\sum_{i=1}^{r} h_i(z(t)) = 1$ for all t, where $h_i(z(t)) \geq 0$ are normalized weights. Some typical T-S exact representations for chaotic systems are shown in [6]. Now we define a T-S fuzzy representation of a chaotic reference model as

$$\dot{x}_r(t) = \sum_{i=1}^{r} h_i(z_r)\{A_{ri} x_r(t) + B_i r(t)\} \tag{2}$$

where $x_r(t)$ is the reference state; and $r(t)$ is a bounded reference signal.

Assumption 1 The reference states are bounded. ■

For the plant (1), we design the input as

$$v(t) = \sum_{i=1}^{r} h_i(z)\{-H_i x(t) + u(t)\} \text{ for } i = 1, 2, \ldots, r. \tag{3}$$

where H_i are the controller gains and $u(t)$ is a new control input to be designed. The resulting closed-loop system is

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)\, h_j(z) \left\{ (A_i - B_i H_j)\, x(t) + B_i u(t) \right\}. \tag{4}$$

The input (3) compensates the mismatching elements between the system matrix of the plant and reference model under the following conditions.

Theorem 1 *Given B_i and the mismatching elements of A_i and A_{ri}. If there exists $\Gamma = \Gamma^T > 0$ such that the following eigenvalue problem (EVP)*

$$\begin{array}{l} \text{minimize } \varepsilon \\ \text{subject to } \Gamma > 0,\ \varepsilon > 0 \\ \left[\begin{array}{cc} \varepsilon I & \left\{ (A_i\Gamma - B_i N_j) - A_{ri}\Gamma \right\}^T \\ \left\{ (A_i\Gamma - B_i N_j) - A_{ri} \right\} & I \end{array} \right] > 0 \end{array} \tag{5}$$

is feasible where $N_j = H_j\Gamma$ for $i, j = 1, 2, \ldots, r$, then $\|A_i - A_{ri}\|^2 < \varepsilon$.

Proof. The proof is omitted due to lack of space. ■

III. Chaotic Reference Model Tracking

We will use an observer-based design to estimate the unknown states of the reference model as

$$\begin{aligned} \dot{\hat{x}}(t) &= \sum_{i=1}^{r} h_i(z) \left\{ A_i \hat{x}(t) + B_i u(t) + L_i (y(t) - \hat{y}(t)) \right\} \\ y(t) &= \sum_{i=1}^{r} h_i(z)\, C_i \hat{x}(t). \end{aligned} \tag{6}$$

Define the estimation error $e(t) = x(t) - \hat{x}(t)$, then we have

$$\dot{e} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)\, h_j(z) \left\{ (A_i - L_i C_j)\, e \right\}. \tag{7}$$

Since the state $x(t)$ in (3) is immeasurable, we replace it by estimation $\hat{x}(t)$, which leads the error system along the modified plant model (4) and reference model (2) has the following form

$$\begin{aligned} \dot{\tilde{x}} &= \sum_{i=1}^{r} h_i(z) \left\{ A_{ri}\tilde{x} + B_i (u(t) - r(t)) \right\} \\ &\quad + \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)\, h_j(z)\, B_i H_j e + \omega \end{aligned} \tag{8}$$

where $\tilde{x}(t) \equiv x(t) - x_r(t)$ (the fact that Assumption 1 and weighting functions are bounded is used); $\omega(t) = \sum_{i=1}^{r} \left\{ h_i(z(t)) - h_i(z_r(t)) \right\} \left\{ A_{ri} x_r(t) + B_i r(t) \right\}$ and is regarded as a bounded disturbance. Using controller $u(t) = r(t) - \sum_{i=1}^{r} h_i(z(t))\, K_i \left\{ \hat{x}(t) - x_r(t) \right\}$ in the system (8), we obtain the closed-loop system

$$\dot{\tilde{x}} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)\, h_j(z) \left\{ (A_{ri} - B_i K_j)\, \tilde{x} + B_i (K_j + H_j)\, e \right\} + \omega. \tag{9}$$

From (7) and (8), an augmented system is given as

$$\dot{\widetilde{\Phi}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)\, h_j(z)\, \widetilde{A}_{ij} \widetilde{\Phi} + \tilde{\omega} \tag{10}$$

where

$$\widetilde{\Phi}(t) = \begin{bmatrix} \tilde{x} \\ e \end{bmatrix},\ \widetilde{A}_{ij} = \begin{bmatrix} A_{ri} - B_i K_j & B_i (K_j + H_j) \\ 0 & A_i - L_i C_j \end{bmatrix},\ \tilde{\omega} = \begin{bmatrix} \omega \\ 0 \end{bmatrix}.$$

For the tracking control of (10), we give the following result

Theorem 2 *The augmented error system (10) has robust performance $\int_0^T \widetilde{\Phi}^T(t)\, Q \widetilde{\Phi}(t)\, dt \le \widetilde{\Phi}^T(0)\, P \widetilde{\Phi}(0) + \frac{1}{\rho^2} \int_0^T \|\omega(t)\|_2^2\, dt$ for a given semi-positive definite symmetric matrix Q if there exists a common matrix $P = P^T > 0$ and such that*

$$\begin{bmatrix} \widetilde{A}_{ij}^T P + P \widetilde{A}_{ij} + Q & P \\ P & -\frac{1}{\rho^2} I \end{bmatrix} \le 0 \tag{11}$$

for all (i, j).

Proof. The proof is omitted due to lack of space. ■

Now we must convert Thm. 2 into an LMI to find the proper gains K_i and L_i which solve the tracking control problem. Assume that

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix},\ Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}.$$

Since $P = P^T > 0$ and $Q = Q^T \ge 0$, we have P_1, P_2 are positive symmetric matrices and Q_1, Q_2 are semi-positive definite matrices. From the assumption on P, Q, inequality (11) and using Schur's complement [8], we have

$$\begin{bmatrix} W_{11} & P_1 B_i (K_j + H_j) & P_1 & 0 \\ (K_j^T + H_j^T)\, B_i^T P_1 & W_{22} & 0 & P_2 \\ P_1 & 0 & -\frac{1}{\rho^2} I & 0 \\ 0 & P_2 & 0 & -\frac{1}{\rho^2} I \end{bmatrix} \le 0 \tag{12}$$

where $W_{11} = A_{ri}^T P_1 + P_1 A_{ri} - K_j^T B_i^T P_1 - P_1 B_i K_j + Q_1$ and $W_{22} = A_i^T P_2 + P_2 A_i - C_j^T L_i^T P_2 - P_2 L_i C_j + Q_2$. Premultiplying and postmultiplying (12) by $\overline{X}_1$ =block-diag$\{X_1, I, I, I\}$ with $X_1 = P_1^{-1}$ and using Schur's complement, we have

$$\begin{bmatrix} \overline{W}_{11} & B_i (K_j + H_j) & I & 0 \\ (K_j^T + H_j^T)\, B_i^T & W_{22} & 0 & P_2 \\ I & 0 & -\frac{1}{\rho^2} I & 0 \\ 0 & P_2 & 0 & -\frac{1}{\rho^2} I \end{bmatrix} \le 0, \tag{13}$$

for all (i, j) where $\overline{W}_{11} = X_1 A_{ri}^T + A_{ri} X_1 - X_1 K_j^T B_i^T - B_i K_j X_1 + X_1 Q_1 X_1$. In attempt to formulate (13) as an LMI problem, we let $M_j = K_j X_1$. Then $\overline{W}_{11} = X_1 A_{ri}^T + A_{ri} X_1 - M_j^T B_i^T - B_i M_j + X_1 Q_1 X_1$. However, The inequality (13) is still not an LMI due to the conflict of K_j in $\overline{W}_{11}$ and M_j. Hence, we will utilize a three step method. In the first step, (5) is solved. In the second step, $\overline{W}_{11} < 0$ is considered. Then we have the following LMI

$$\left[\begin{array}{cc} X_1 A_{ri}^T + A_{ri} X_1 - M_j^T B_i^T - B_i M_j & X_1 Q_1^{\frac{1}{2}} \\ Q_1^{\frac{1}{2}} X_1 & -I \end{array} \right] < 0. \quad (14)$$

From solving (5) and (14), we obtain Γ, N_j, X_1, M_j and therefore the compensation, new controller gains H_j, K_j, respectively. Substituting these parameters into (13) and letting $O_i = P_2 L_i$, inequality (13) becomes an LMI. In the third step, we obtain observer gains L_i from solving for (13).

IV. Numerical Simulations

To verify the theoretical derivation, we apply the above method to the tracking control of the chaotic Chua's circuit [9] with immeasurable states. The reference model is governed by the following system equations:

$$\begin{aligned} \dot{x}_{r1}(t) &= \alpha(-x_{r1}(t) + x_{r2}(t) - f(x_{r1}(t))) \\ \dot{x}_{r2}(t) &= x_{r1}(t) - x_{r2}(t) + x_{r3}(t) \\ \dot{x}_{r3}(t) &= -\beta x_{r2}(t) \end{aligned} \quad (15)$$

where $f(x_{r1}(t)) = g_a x_{r1}(t) + 0.5 (g_a - g_b) (|x_{r1}(t) + 1| - |x_{r1}(t) - 1|)$. Using the T-S fuzzy modeling method in [4], the reference system (15) is represented as the fuzzy rules

Reference rule i: IF $x_{r1}(t)$ is F_i THEN $\dot{x}_r(t) = A_{ri} x_r(t)$

for $i = 1, 2$. The system matrices are

$$A_{r1} = \left[\begin{array}{ccc} (d_r - 1)\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{array} \right], \; A_{r2} = \left[\begin{array}{ccc} -(d_r + 1)\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{array} \right]$$

with fuzzy sets $F_{r1} = \frac{1}{2}\left(1 - \frac{\phi(x_{r1})}{d_r}\right)$, $F_{r2} = 1 - F_{r1}$ where $d_r = \sup_{x_{r1} \in \Omega} |\phi(x_{r1})|$ and

$$\phi(x_{r1}) = \begin{cases} f(x_{r1}) / x_{r1}, & \text{for } x_{r1} \neq 0 \\ g_a, & \text{for } x_{r1} = 0. \end{cases}$$

For the reference model, the parameters $(\alpha, \; \beta, \; g_a \; g_b) = (9 \;\; 14.29 \;\; -1.43 \;\; -0.71)$ and initial conditions $(2.53 \;\; 0.00 \;\; -4.81)$. For the plant, we assume that only state $x_1(t)$ of the reference is measurable. The plant model is given as

$$\begin{aligned} \dot{x}_1(t) &= \Delta\alpha(-x_1(t) + x_2(t) - f(x_1(t))) + v(t) \\ \dot{x}_2(t) &= x_1(t) - x_2(t) + x_3(t) \\ \dot{x}_3(t) &= -\beta x_2(t) \end{aligned} \quad (16)$$

where $\Delta\alpha \equiv \alpha + \delta$ and δ denotes an arbitrary deviation in the parameter α; and $f(x_1(t)) = g_a x_1(t) + 0.5 (g_a - g_b) (|x_1(t) + 1| - |x_1(t) - 1|)$. The fuzzy rules for the plant are

Plant rule i: IF x_1 is F_i THEN $\dot{x} = A_i x + Bv$

$$y = Cx$$

for $i = 1, 2$. The system matrices are

$$\begin{aligned} A_1 &= \left[\begin{array}{ccc} (d - 1)\Delta\alpha & \Delta\alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{array} \right], \; A_2 = \left[\begin{array}{ccc} -(d + 1)\Delta\alpha & \Delta\alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{array} \right] \\ B &= \left[\begin{array}{ccc} 1 & 0 & 0 \end{array} \right]^T, C = \left[\begin{array}{ccc} 1 & 0 & 0 \end{array} \right]. \end{aligned}$$

with fuzzy sets $F_1 = \frac{1}{2}\left(1 - \frac{\phi(x_1)}{d}\right)$, $F_2 = 1 - F_1$ where $d = \sup_{x_1 \in \Omega} |\phi(x_1)|$ and

$$\phi(x_1) = \begin{cases} f(x_1) / x_1, & \text{for } x_1 \neq 0 \\ g_a, & \text{for } x_1 = 0. \end{cases}$$

For simulations here, we let $\delta = 1$. In Fig. 1, we depict the observer-controller based Chua's circuit (16). Here we illustrate the tracking errors in Fig. 2. The 3-dimensional tracking performance is shown in Fig. 3 (reference plant dotted lines).

V. DSP-Based Experiments

Based on the numerical simulations above, we carry out DSP-based experiments and show in Figs. 4 thru 6 the oscillocope images of the results for plant states $x_1(t)$, $x_2(t)$, $x_3(t)$ tracking reference states $x_{r1}(t)$, $x_{r2}(t)$, $x_{r3}(t)$, respectively.

VI. Conclusions

In this paper, we have extended the fuzzy tracking control to chaotic reference models with mismatched parameters and immeasurable states. Results of numerical simulations and DSP-based experiments on the well-known Chua's circuit shows the validity of the approach.

References

[1] E. Ott, C. Grebogi and J. A. Yorke, "Controlling chaos," *Phys. Rev. Lett*, vol. 64, pp.1196-1199, 1990.

[2] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern.*, vol. 15, no. 1, pp. 116-132, 1985.

[3] K. Tanaka, T. Ikeda, and H. O. Wang, "A unified approach to controlling chaos via an LMI-based fuzzy control system design," *IEEE Trans. Circuits Syst. I*, vol. 45, no. 10, pp. 1021-1040, 1998.

[4] K.-Y. Lian, T.-S. Chiang, C.-S. Chiu, and P. Liu, "Synthesis of fuzzy model-based design to synchronization and secure communication for chaotic systems," *IEEE Trans. Syst. Man Cybern. Part B.*, vol. 31, pp. 66-83, 2001.

[5] H. J. Lee, J. B. Park and G. Chen, "Robust fuzzy control of nonlinear systems with parametric uncertainties," *IEEE Fuzzy Syst.*, vol. 9, pp. 369-379, 2001.

[6] K.-Y. Lian, C.-S. Chiu, and P. Liu, "LMI-Based Fuzzy Chaotic Synchronization and Communications," *IEEE Trans. Fuzzy Syst.*, vol. 9, no, 4, pp. 539-553, 2001.

[7] C.-S. Tseng, B.-S. Chen, H.-J. Uang, "Fuzzy tracking control design for nonlinear synamic systems via T-S fuzzy model," *IEEE trans. Fuzzy Syst., vol. 9, pp. 381-392, 2001.*

[8] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory.* Philadelphia, PA:SIAM, 1994.

[9] L. O. Chua, M. Komuro, and T. Matsumoto, "The double scroll family, part I: Rigorous proof of chaos," *IEEE Trans. on Circuits. Syst.*, vol. 33, pp. 1072-1096, 1986.

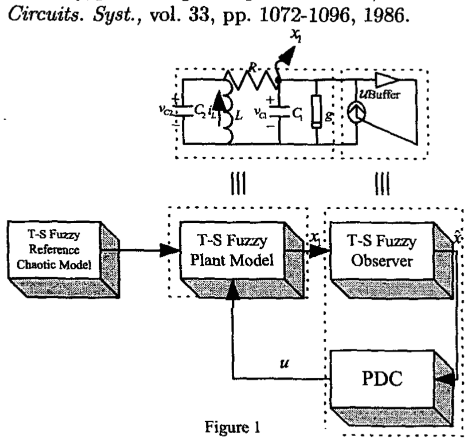

Figure 1

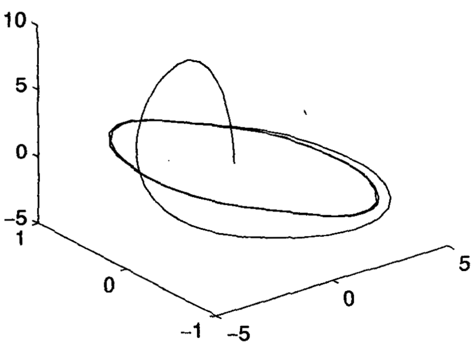

Figure 2

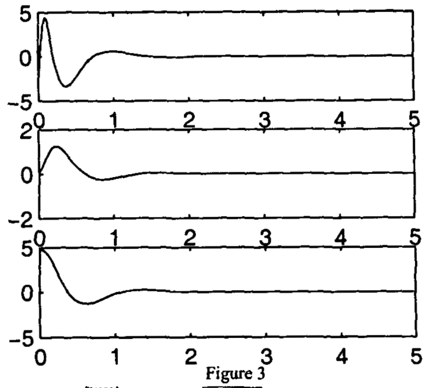

Figure 3

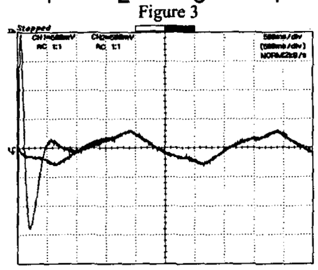

Figure 4

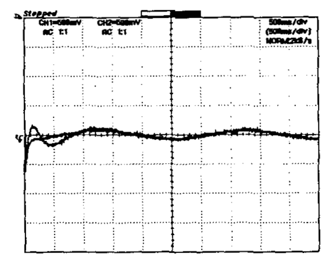

Figure 5

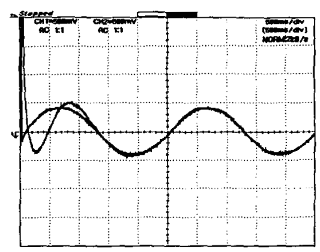

Figure 6